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A theoretical proposal for the design of a photoretinoscopic apparatus

Abstract

A theoretical proposal for the design of a photoretinoscopic apparatus

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Master of Science in Vision Science

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Niles Roth, O.D., Ph.D.

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A Theoretical Proposal for the Design of a Photoretinoscopic Apparatus

By: Jeffrey R. Urness

Advisor: Niles Roth O.D., Ph.D.

May 17, 1985

Submitted for Fulfillment of Opt 592

approved
Niles Roth

TABLE OF CONTENTS

	<u>Page</u>
Acknowledgements	-i-
Introduction	-1-
Literature	-2-
Design Proposal	-9-
Design Principles	-9-
Laboratory Prototype Design	-13-
References	-16-
Appendix A Analytical Proofs	-17-
Appendix B Computer Programs and Output	-20-
Appendix C Laboratory Prototype Schematic	-24-

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INTRODUCTION

It is well known that undetected refractive errors in young children can incite abnormal visual and/or visual motor development. Early detection and remediation, in many cases, would prevent this abnormal development.¹ The major difficulty in providing remedial care for children who possess significant refractive errors is detection of the abnormalities.

The nature of the pediatric population is such that subjective testing, if possible, is often invalid and/or unreliable. This leaves parental case history and objective examination as the major tools available with which to detect refractive errors occurring in young children. Traditionally, familial observation and history have been greatly relied upon to detect vision problems in children. This reliance has resulted from the lack of efficiency and effectiveness in traditionally utilized objective test techniques. The shortcomings of these customarily used procedures are due, in great part, to the fact that many techniques are adaptations of adult designed test procedures. Their need for patient cooperation, attention and understanding often exceeds the abilities most young children possess. Therefore, if valid and reliable objective findings are to be gathered from the pediatric patient, an experienced and skilled clinician must be enlisted or new objective test procedures will need to be developed.

New objective testing procedures, to be effective, should minimize the effect of poor patient cooperation, attention and understanding. One method which will achieve this is photographic recording of procedure endpoints. Specific to my concern is the possible application of photographic recording to procedures designed for determining refractive status in young children.

LITERATURE REVIEW

Photographic technique used as a probe to evaluate the status of the visual system is not a new concept. Maddox (1902) used photography to attempt evaluation and quantification of strabismic deviations.⁶ Graham and Naylor (1957), Weekers (1963), Joachim and Gilson (1964), Breitenmoser and Wurth (1970) and Jones and Eskridge (1970) all photographed the Hirschberg test and attempted its quantification.⁶

Photographic techniques have also been applied to procedures designed to determine ocular refractive status. I will refer to these procedures collectively as photorefraction. Application of photographic techniques to testing utilized in evaluation of refractive status is a relatively new concept, when compared to methods exploring the status of binocular posture.

Howland, a pioneer in the development of photorefractive procedures, was one of the first to report on a system designed to study refraction at distance (1974).³ His apparatus consisted of a Nikon 35mm SLR camera body fitted with a 55mm f/1.2 lens. This stock camera system was modified to incorporate a sector cylinder lens/fiber optics attachment. It is this attachment, designed by Howland, which makes possible the quantification of ocular refractive status. The system's principle of operation is as follows: Light is delivered to the eye from the central fiber optic probe. Entering light is then refracted by the eyes' optical components, being directed to the image plane located in the vicinity of the retina. The clarity of this image upon the retina is a function of the pupil diameter and refractive status of the test eye. Light forming this retinal image is then reflected off the same membrane, returns through the ocular media, and is refracted a second time

during its exodus. Upon emerging from the test eye, the light has assumed an exit vergence dictated by the ocular refractive state at the time of the light's passage out of the eye. This light energy is then captured by camera lens at a distance of two meters from the patient. The diameter of the image at the film plane is a function of the retinal image size, ocular refractive state, pupil diameter, working distance and lens power of the photographic recording system. The sector cylinder lens attachment intercepts the light output before it reaches the film plane, imparting meridional vergence changes, as well as producing four separate optical paths which result in a Schiener's image. This Schiener's image is a star shaped resultant of the four elongated images formed from the retinal reflection. The length of these elongated images is a function of the refractive state, pupil diameter and camera lens power. It is the length of these "star arms" that, when interpreted, provide the endpoint refractive state of the test eye relative to the camera recording system. With this design, Howland can determine the refractive error of a given eye in two meridians using one exposure. To determine the sign of the error, relative to the camera, Howland uses successive exposures performed with red and blue fiber optic probes. Using these probes in succession, astigmatic errors are detectable as differential "star arm" lengths in one or the other photo. Spherical errors appear as uniform four point star images. To recover the endpoint information, the 35mm film is processed and its images magnified 20 diameters at which point the length of the "star arms" can be accurately measured. The refractive error is estimated based on the length of the images and the diameter of the pupil. Advantages of this system: potential endpoint accuracy, portability,

simultaneous binocular record. Disadvantages of this system: multiple exposures to determine endpoint refractive state, processing and enlargement of images to determine endpoint refractive state, significant skill level needed to interpret records, undetermined accuracy. Though Howland's system possesses some shortcomings, its positive characteristics make the system a significant advancement in the currently available testing methods.

A significantly different system, patented by Grolman, was reported in 1975 by Guyton.² Grolman's system consists of an object array of fiber optic point sources located at different dioptric distances from the test eye. These point sources are imaged on the patient's fundus where the image of greatest clarity identifies the conjugate object point within the array. These retinal images become objects, emitting light outward through the test eye's ocular refractive elements and into the photographic recording system. This light information is then recorded on polaroid film and instantly developed for endpoint determination. The "clearest" image on the film identifies the dioptric magnitude of the refractive status of the test eye. Advantages of this system: potential accuracy, single photograph to determine endpoint, easily detected astigmatism. Disadvantages of this system: accuracy of system is yet undetermined.

Kaastrup and Woods (1978), senior optometry students reported on an apparatus similar to that of Grolman.⁸ This system consists of a +5.50 diopter lens and five light guide point sources arranged in a Badal configuration. The five sources are located at five dioptric distances from the test eye, ranging from +2.00 diopters to -2.00 diopters in one diopter steps. Light from the five point sources is imaged on the test eye's fundus.

The "clearest" image denotes the object conjugate with the fundus. These five images in turn act as objects sending light back out of the eye to be received and recorded with a camera and polaroid film recording system. The endpoint is easily and reliably determined with minimal skill. This study reported results of schematic eye testing only. Advantages of system: linear relation between object loci and "ocular" vergence values, constant image size and location on fundus for a given eye, potential accuracy and ease of endpoint interpretation; endpoint determined from single exposure, astigmatic errors are detected in one photographic record. Disadvantages of this system: not portable, limited range of refractive error measurement, and short working distance.

In 1979, Kaakinen reported yet another approach.⁶ Kaakinen's procedure utilizes a canon EF 35mm SLR camera body in conjunction with an FD 100mm f/2.8 lens. The delivery system is a sunpak GX 17 electronic flash mounted coaxially in a horizontal fashion just superior to the objective lens and in the same vertical plane. The principle used in determination of refractive status is as follows: light entering the eye is brought to a circular image on the fundus; the image diameter a function of refractive error and pupil diameter. This light is then reflected back out of the eye. For refractive errors between +4 diopters and -3 diopters light from all portions of the eye's exit pupil are received by the recording system. For refractive errors outside this range light from a certain area of the eye's exit pupil does not enter the recording system, resulting in the presence of a dark crescent in the superior pupil for myopia and the inferior pupil for hyperopia. Advantages of this system: portability, binocular information regarding

refractive state and binocular posture attained in one exposure, minimal cost of apparatus construction, relative ease in interpretation of endpoint record. Disadvantages of this system: large refractive dead zone, endpoint can only be placed within a range of refractive errors, refractive status of only the vertical meridian is determined, possibility of results being interpreted as false negatives.

In 1980, Kaakinen reported a modification to his original apparatus and procedure, this being the addition of a second light source mounted in horizontal meridian adjacent to the lens objective in a vertical orientation.⁷ This addition makes possible the detection of large astigmatisms by producing pupillary crescents in both the vertical and horizontal meridians.

A modified Howland system, termed isotropic photorefracton, was reported by Howland in 1983.⁴ Removing the four sector cylinder lenses from the original attachment and altering the original procedure, Howland uses three exposures to determine an estimated endpoint refractive state. The three exposures are performed with the lens system focused behind, at and in front of the test eye. The change in shape and size of the images between photographic records is used to determine the estimated refractive error. Howland states that this new method is effective in determining principal meridians, but must be combined with his original system to accurately estimate the magnitude of a given eye's refractive error. If used alone, this system should only be applied as a screening test for referral purposes.⁴

As with his original system, Howland is to be commended for his extensive theoretical work. Advantages of this system: provides records in all

meridians, allowing determination of principal meridians. Disadvantages of the system: must be used in conjunction with original system for other than screening applications, requires multiple exposures to determine endpoint, significant skill level required to perform and interpret procedure, record must be enlarged to determine refractive error estimate.

The most recent report of a different apparatus used to photographically detect refractive errors was reported by Molteno and Associates (1983).⁹ This system is very similar in principle to that reported by Kaakinen. Molteno's system, named the Otago photoscreener, uses an annular light delivery system which is located in the plane of and symmetrically surrounding the objective lens. This annular light source also acts as the limiting aperture of the optical system. It is this factor that is exploited in determination of refractive state endpoint. The operating principle of this system is as follows: light enters the eye and is reflected off three structures at the fundus: vitreoretinal interface, bruch's membrane and the sclera. These reflections consist of predominantly different wave lengths white (combination of wavelengths), yellow and red respectively. If the eye's refractive posture is such that it is conjugate with the light source, all light reflected out of the eye will be imaged atop the annular strobe resulting in a dark pupil. In some cases, the pupil will appear dark red. This results from the fact that the sclerally reflected light is most diffuse, therefore its image sometimes overlaps the light source. As the eye's refractive state varies more and more from conjugacy, the magnitude of blur increases at the annular source plane. As the blur progressively increases, changing wavelengths enter the photographic recording system. This is from light reflected from Bruch's

membrane and the vitreoretinal interface. By noting the color of the pupillary reflex in the photographic records, one can estimate the test eye's refractive state relative to the objective lens plane. Advantages of this system: information regarding binocular refractive state is derived from one photograph, procedure is easily performed, portable one piece apparatus. Disadvantages of this system: requires significant skill level for proper endpoint interpretation, errors greater than 12 diopters can go undetected, large refractive error dead zone, specific endpoint accuracy is questionable although refractive ranges are reliably estimated.

In summary, Howland's original method, combined with his modification, currently provides the greatest potential accuracy, yet this system is specially manufactured, requires enlargement of original photographic record and significant skill level to interpret and determine refractive error endpoint. Grolman's design appears potentially accurate, yet bulky and untested. Kaastrup and Wood's system shows promise, but is very basic, bulky and of limited endpoint range. Kaakinen's procedure is easily performed and interpreted, yet is of questionable accuracy and sensitivity. Molteno's system is easily operated and relatively easy to interpret, yet is lacks sensitivity, specificity and accuracy. This system is good for screening out large ametropias. Although all of these systems have outstanding characteristics none of them combines, what I believe are four critical traits: specificity, sensitivity, ease of interpretation and operation, and portability at a reasonable cost.

DESIGN PROPOSAL

It is apparent from the literature review that the optimal photographic refractive test procedure has not yet been developed. The optimal system should be characterized by high specificity and sensitivity, portability, ease of operation, instantaneous single exposure binocular record and ease in interpretation of endpoint photographic record; all at a reasonable cost. The following design is expected to further the effort in developing an optimal photorefractive system.

The proposed system design is superficially similar to that of Grolman's in the use of fiber optic "point" sources forming the object test array.² Kaastrup and Wood's research provides the encouraging foundation for the application of a Badal light delivery system, which is basic to the proposed design.⁹ Although encouraging, Kaastrup and Wood's research was empirical in nature.⁹ Their laboratory system, although of primitive design, exhibited high levels of both validity and reliability when used to test schematic standards.⁹ Limited testing range, short working distance and bulky delivery system are factors which pose barriers to the development of Kaastrup and Wood's system into a clinical prototype appropriate for pediatric evaluation. These are three barriers that will be removed by the proposed design. This new system design will be named "A Three Element Confocal Badal Photoretoscope" (TECBP).

DESIGN PRINCIPLES

As with conventional retinoscopic procedures, the TECBP system is composed of a light delivery/imaging component and a light receiving/observing

component (Appendix C). For the TECBP, the light delivery/imaging component consists of a multi-point stimulus object array and a three element confocal Badal lens systems. The light receiving/observing component consists of a beam splitter, receiving lens and photographic recording film. For this system, the receiving lens has a small aperture and is set conjugate to infinity.

More specifically, the light delivery system (Appendix C) combines a multi-point object array with a three element confocal Badal lens system to provide a discrete range of input vergences at the eye. These input vergences vary in a linear fashion as a function of object position in the source array (Appendix A). This linear relationship will be exploited to extend the system's range, while reducing the number of fiber optic objects within the physical array. The range extension will be attained by using three position settings for the object array. An acceptable ocular input vergence range is achieved by combining the system's linearity with a second critical feature, vergence magnification. This characteristic is a product of the three element confocal lens design. Combining vergence linearity and magnification, the system's object array dimension can be greatly reduced, while at the same time providing the desired ocular input vergence range. While the excessive bulk of the Kaastrup/Wood's system is greatly reduced with the reduction of the axial object array dimension, the proposed system would still be prohibitively long if a linear arrangement of the elements were utilized. Therefore the delivery system of a clinical prototype would incorporate a series of plane mirrors to maintain the appropriate optical path lengths while reducing the physical dimensions to acceptable values. This "folding" of the delivery system will not be incorporated until production of a clinical prototype is undertaken.

The third and final feature of the TECBP delivery system is its clinically appropriate working distance. The working distance of the system is strictly function of the output lens power, the reason being that the system's secondary focal point is coincident with the output lens' secondary focal point and the system's final focal point must be coincident with the ocular nodal point to maintain its Badal property. As a result, the system's working distance is approximately equal to the second focal length of the output lens. By varying the power of the output lens, the system's working distance can be adjusted.

Integrating a three element confocal design with the Badal principle removes the barriers present with the Kaastrup/Woods system, while maintaining the Badal feature of constant image size in the eye (Appendix A). This feature is critical to the receiving component of the TECBP system and also the endpoint interpretation of the photographic record. Another feature affecting the photographic record, is that of uniform flux density received from all points in the object array. This characteristic results from the fact that the TECBP lens design closely approximates a Maxwellian View configuration and that for a Maxwellian View optical system the flux density of an image is not a function of the object distance.¹⁰ Therefore, all input imagery of the TECBP system is presented to the eye with uniform light flux density, assuming all object sources are uniform in emitted areance (luminance).

Once the light delivery system has input the multiple vergence light energy to the eye, the light is refracted by the ocular elements. Only one

source object within the array, corresponding to one ocular input vergence, is found conjugate to the retinal plane. All other source objects, within the array, are conjugate to planes anterior and posterior to that of the retinal plane. This results in a composite retinal image array, corresponding to the object array, consisting of source images at various stages of defocus, with the exception of the source image which is conjugate to the retina. The relative clarity of the non conjugate images will be a function of pupil diameter, refractive status and media "clarity". Having formed the retinal image array, corresponding to the object array, the light delivery/imaging component of the TECBP system has completed its function. This retinal image array now acts as the object for the receiving/observing component of the TECBP system. Light leaving the retinal image array exits the eye and is refracted as it exists. It is then intercepted by the objective of the receiving lens system, and transformed into an image of varying clarity, at the photographic recording film plane. Using a small receiving aperture (high f number), the effects of varying entrance vergence are minimized. Any effect that occurs will be experienced uniformly by all points in the film plane. Having exposed the film, it is then processed such that the endpoint can be estimated. The endpoint is determined by locating the sharpest image within the array, noting the dioptric value of its conjugate source object. Astigmatic errors will be detectable via elongation of nonconjugate images.² It is not known at this writing if the magnitude of these astigmatic errors will be easily estimated.

It is to be noted that all previously stated principles assume that the separation between the ocular nodal point and principal plane is negligible and they are, therefore, treated as coincident.

In summary, a confocal Badal lens system combined with a fiber optic object array will serve as the light delivery/imaging component, while a 35mm SLR camera with polaroid back will serve as the receiving/observing component of the TECBP system. Linearity of vergence, vergence magnification, uniform flux input and adequate working distance are all key features of this design.

LABORATORY PROTOTYPE DESIGN

The laboratory TECBP system design was derived with the assistance of computer analysis. The analysis was performed on an expanded Commodore microcomputer VIC 20. Two programs, written in basic, were used to aid in the determination of the TECBP system parameters (Appendix B). The first program was designed to systematically test the many combinations of lens powers to determine which combinations would provide the desired ocular input vergence range, object array dimension and dioptric scale factor. Once the lens powers were optimized, a second program was used to lay out the axial locations of the fiber optic object sources corresponding to a specific discrete range of ocular input vergences. This program was also utilized to determine the fiber optic diameters and paraxial locations needed to produce a retinal image array subtending three degrees at the macula, relative to the ocular nodal point. Prior to using these two programs, a working distance was chosen, thereby determining the output lens power. Upon the advice of Dr. Paul Kohl, director of the pediatric clinic Pacific University College of Optometry, a working distance of one-half meter was chosen, thereby determining a +2 diopter output lens. Possible lens element combinations were derived based on the following arbitrarily chosen parameters: ocular input vergence test range 30 diopters

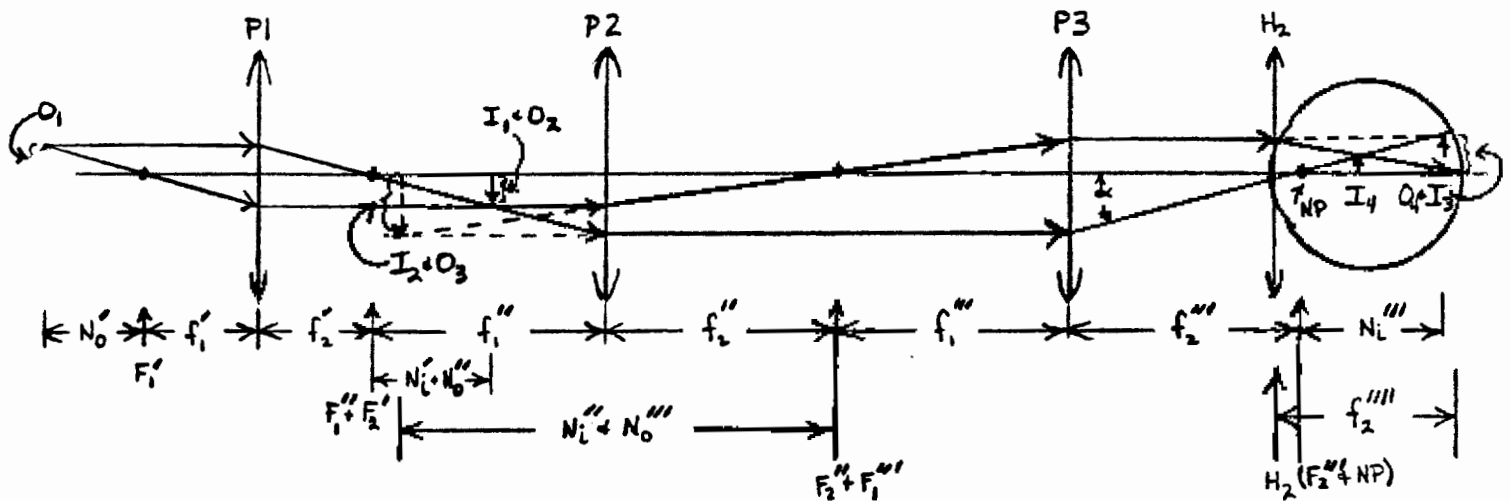
(+15 to -15 diopters), axial dimension of object array 0.15 meters maximum, output lens power +2 diopters. The following lens powers were determined to produce the most acceptable system for the present purposes: input lens +13.375 diopters, intermediate lens +1.875 diopters, output lens +2.0 diopters. Utilizing these elements, the actual object array axial dimension needed to produce an ocular input range of +15 diopters to -15 diopters was 0.147 meters. This, in turn, produced a dioptric scale factor of 2.0 diopters/centimeter. Knowing the dioptric parameters of the system, the second computer program was used to derive the fiber optic diameter and distance from the axis. The results are as follows: fiber optic diameter 0.5mm, distance from axis 1.6mm. The object array will consist of 10 fiber optic source objects symmetrically placed around the system's optic axis at their respective axial positions. The separation between successive objects in the array will correspond to one diopter at the focal plane of the output lens. Light energy emitted by the fiber optic source objects will be provided by an electronic vivitar 283 photographic flash. The object array at any given distance from the input lens will produce a discrete 10 diopter ocular input vergence range. By providing three different settings for the object array, a 30 diopter range can be tested. For standard testing, the array will be placed in the central position, given a range of +5 to -5 diopters. For refractive errors outside this range the array will be shifted to the appropriate location as indicated by preliminary photos. The receiving component of the TECBP system will be a Nikon FM 35mm SLR camera body with polaroid back and 80 to 200mm f/4.5 AI Nikkor zoom lens. This zoom lens will allow easy empirical testing of different focal length settings. For the

laboratory system the delivery component and the receiving component will have optic axis at approximately 90 degrees to one another. The optic axis of the receiving component will be rendered coincident with the axis of the delivery system between the beam split and the test eye. This will be achieved using a half silvered mirror Figure 1, (Appendix C).

The planned sequence of testing and standardization in the laboratory setting will proceed as follows: simulation of various refractive errors using schematic eyes, monocular measurements of adult human subjects and comparison of these values to those obtained via traditional clinical methods. If these results show promise when compared to theoretical and clinical endpoints, we will begin construction of a clinical prototype designed to be handheld and able to produce a binocular record on young children.

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Using Newtonian optical relationships, the following mathematical proofs will be given to validate the two stated characteristics of the TECBP system. These characteristics are constant retinal image size for a constant source object size and linearity of relationship between ocular input vergences and source object location. Note: these proofs assume the ocular principal plane and nodal point are coincident. The error induced by this assumption is believed, by this author, to be insignificant, although its exact magnitude is not addressed at this time.

LEGEND

Capital letters O and I indicate object and image heights respectively. Their subscripts denote which lens element the given object or image is associated with.

N_o and N_i indicate object and image distances relative to the focal points. Primes denote which lens element the distance are relative to.

NP is the ocular nodal point.

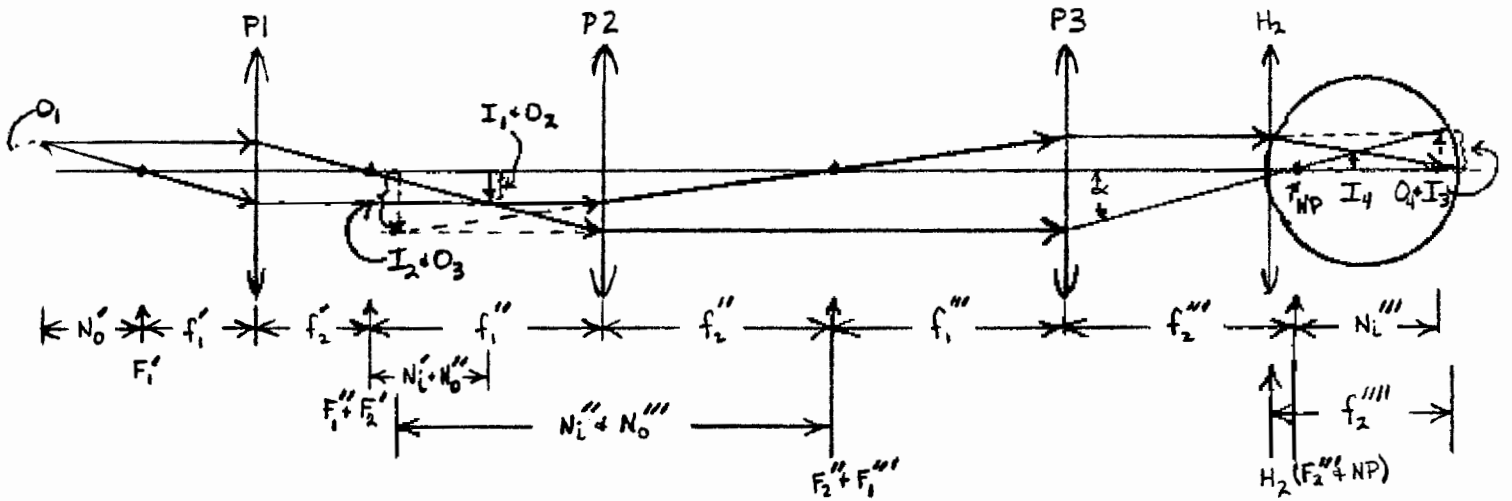
H_2 is the ocular principal plane.

P1 is the first element, the input lens.

P2 is the second element, the intermediate lens.

P3 is the third element, the output lens.

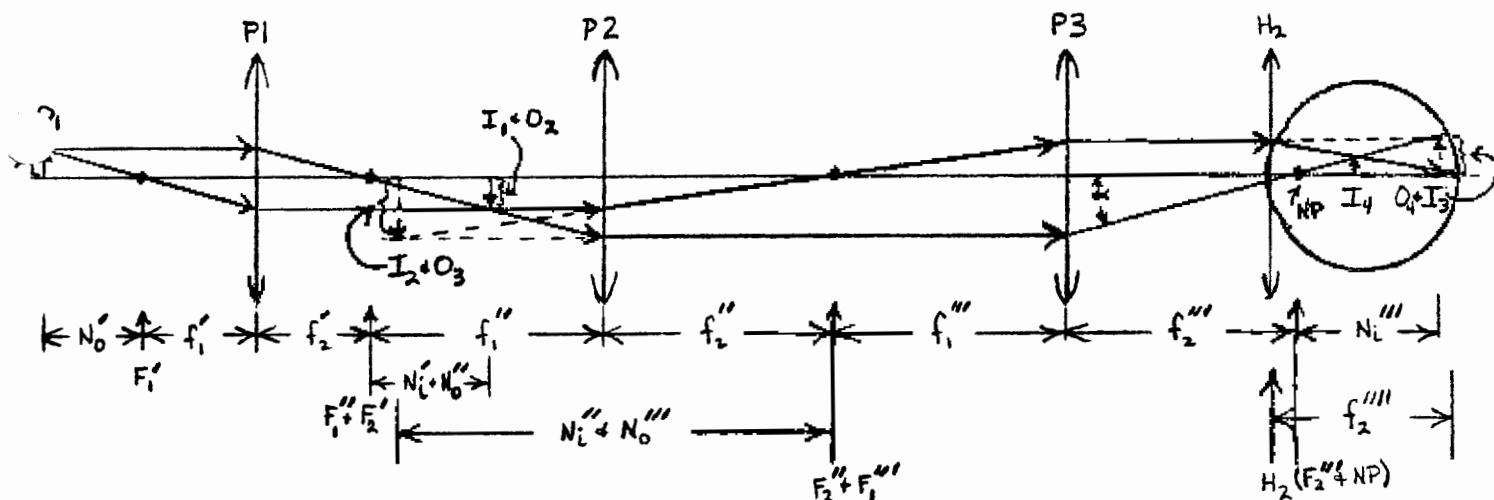
is the angular subtense of the retinal image relative to the ocular nodal point.



Proof of Constant Retinal Image Size
resulting from a constant source object size

- a. $N_i' = f_1' * f_2' / N_0'$ and $N_0'' = N_i'$ therefore $N_0'' = f_1' * f_2' / N_0'$
- b. $N_i''' = f_1''' * f_2''' / N_0'' = f_1''' * f_2''' * N_0' / f_1' * f_2'$ and $N_0''' = N_i''$
- c. therefore $N_0''' = f_1''' * f_2''' * N_0' / f_1' * f_2'$
- d. $N_i'''' = f_1'''' * f_2'''' / N_0''' = f_1' * f_2' * f_1''' * f_2''' / f_1'' * f_2'' * f_2'' * N_0'$
- e. regrouping $N_0' = f_1' * f_2' * f_1''' * f_2''' / f_1'' * f_2'' * N_i''''$
- f. $N_i'''' = I_3 / \tan$ therefore $N_0' = f_1' * f_2' * f_1''' * f_2''' * \tan / f_1'' * f_2'' * I_3$
- g. also $N_0' = f_2' * O_1 / O_2$ and $O_2 = f_2''' * I_3 / f_2''''$
- h. therefore $N_0' = f_2' * O_1 * f_2'''' / f_2''' * I_3$
substitution for N_0' in line f
- i. $O_1 * f_2' * f_2'''' / f_2''' * I_3 = f_1' * f_2' * f_1''' * f_2''' * \tan / f_1'' * f_2'' * I_3$ factoring line i
- j. $O_1 = (f_1' * f_1''' / f_1'') * \tan$ setting $K = (f_1' * f_1''' / f_1'')$
Since the tangent function is linear for small angles,
- k. $O_1 = (K + \text{constant}) *$

Therefore angular subtense of the retinal image, relative to NP,
is a linear function of source object size.



Proof of Linear Relation for Ocular Input Vergence
as a Function Source Object Location

- Using line e from the previous proof
- $N_0' = f_1' * f_2' * f_1''' * f_2''' / f_1'' * f_2'' * N_1'''$
- $V_4 = \text{ocular input vergence} = 1/N_1'''$
- therefore $N_0' = V_4 * (f_1' * f_2' * f_1''' * f_2''' / f_1'' * f_2'')$
- Setting $C = (f_1' * f_2' * f_1''' * f_2''' / f_1'' * f_2'')$
- $N_0' = V_4 * C$ and $N_0 = V_4 * C$

Therefore, ocular input vergence varies linearly as a function of source object location

Appendix B

LENS ELEMENT OPTIMIZATION PROGRAM FOR DESIGNING THE TECBP SYSTEM

```

5 REM: BADAL SEARCH: PROGRAM TO CALCULATE OPTIMUM DIOPTRIC COMPONENTS...
6 REM:...ORIGINATED BY JEFFREY R URNESS, MAY, 1985.
10 READ P1,P2,P3,O1,IN,C
20 O1=O1:P4=P1
30 P1=P4
40 IF (P1>40) THEN 170
45 FOR Z=1 TO 20: PRINT "P1="P1: NEXT
50 O1=O1: V1=0: V4=0: M=0: IS=0
55 IF (V4>15) THEN 125
70 F1=P1^-1: F2=P2^-1: F3=P3^-1: W1=O1^-1: V1=P1+W1: W2=V1/(1-(F1+F2)*V1)
75 FOR T=1 TO 20: PRINT "O1="O1: NEXT
80 V2=P2+W2: W3=V2/(1-(F2+F3)*V2): V3=P3+W3: V4=V3/(1-(F3)*V3): O1=O1-IN: M=M+1
85 FOR D=1 TO 20: PRINT "V4="V4: NEXT
90 IF (V1<>0) THEN 110
100 V1=V4
110 IF (V1<=-15 AND V4>=-100 AND M<C) THEN 60
120 GOTO 150
125 O1=O1+IN
130 VR=V4-V1: DS=- (O1-O1): DS=VR/OS: FOR K=1 TO 50: PRINT "DS="DS: NEXT
135 IF (DS>250 AND DS>15) THEN 170: IS=DS
140 IF (DS<=205 AND DS>=200) THEN 190
150 P1=P1+0.125
160 GOTO 40
170 P2=P2+0.125
175 RS=1 TO 25: PRINT "P2="P2: NEXT
180 IF (P2<10) THEN 30
185 O1=O1+IN
190 PRINT "O1="O1: PRINT "V4="V4: PRINT "O1="O1: PRINT "V1="V1: PRINT "OS="OS
200 PRINT "VR="VR: PRINT "DS="DS: PRINT "P1="P1: PRINT "P2="P2: PRINT "P3="P3
210 STOP
220 DATA 13.375,1.875,2,-0.001 ,0.025,50

```

SOURCE OBJECT LOCALIZATION PROGRAM FOR
ASSISTING IN DESIGN OF THE TECBP SYSTEM

```

140 OPEN4,4:CMD4
150 PRINT"VARIABLES O1=REAL OBJECT DISTANCE O2=SECOND ELEMENT OBJECT DISTANCE"
160 PRINT"O3=THIRD ELEMENT OBJECT DISTANCE I1=FIRST ELEMENT IMAGE DISTANCE"
170 PRINT"I2=SECOND ELEMENT IMAGE DISTANCE I3=THIRD ELEMENT IMAGE DISTANCE"
175 PRINT"I4=FINAL IMAGE DISTANCE FROM TEST EYE"
180 PRINT"P2=POWER ELEMENT 2 P3=POWER ELEMENT 3"
185 PRINT"W1=VERGENCE O1 W2=VERGENCE O2 W3=VERGENCE O3"
190 PRINT"V1=VERGENCE I1 V2=VERGENCE I2 V3=VERGENCE I3 V4=VERGENCE I4"
195 PRINT"OA=REAL OBJECT SIZE IA=1ST IMAGE SIZE IB=2ND IMAGE SIZE IC=3RD IMAGE S
IZE."
200 PRINT"A=IC/I4 ANGULAR SUBTENCE OF IC AT THE EYE. V=VERGENCE AT EYE'S NODAL P
T."
210 PRINT"V=1/(P3-I3)"
235 PRINT4:CLOSE4
240 OPEN4,4:CMD4
250 PRINT"V4="::INPUTV4
251 P1=13.375
252 P2=1.875
253 P3=2.000
254 A=1.5000
255 VR=-V4
256 I4=VR*-1
257 PRINTV4
258 F1=P1*-1
259 F2=P2*-1
260 F3=P3*-1
265 V3=VR/(1-(F3)*VR)
270 W3=V3+P3
275 V2=W3/(1-(F2+F3)*W3)
280 W2=V2+P2
285 V1=W2/(1-(F1+F2)*W2)
290 W1=V1+P1
295 O1=W1*-1
300 O1=-O1
305 PRINT"O1="O1
310 PRINT"P1="P1
315 PRINT"P2="P2
320 PRINT"P3="P3
325 PRINT"I4="I4
330 PRINT"V4="V4
335 IC=TAN(A/180)*I4
340 IB=IC*V3/W3
345 IA=IB*V2/W2
350 OA=IA*V1/W1
351 O=IC/I4
355 A=(ATN(O))*180
360 PRINT"OA="OA
365 PRINT"IC="IC
37 PRINT"A="A
37 STOP

```

Results for a 30 diopter input vergence range is printed out on the following pages.

V4=-15
 O1=-.0010699
 P1= 13.375
 P2= 1.875
 P3= 2
 I4=-.0666667
 V4=-15
 OA=-1.83546179E-03
 IC= 1.7457E-03
 A= 1.5

PROGRAM OUTPUT
 V4=-10
 O1=-.0256354266
 P1= 13.375
 P2= 1.875
 P3= 2
 I4= .1
 V4=-10
 OA=-1.83546179E-03
 IC= 2.61859216E-03
 A= 1.5

V4=-5
 O1=-.0502008909
 P1= 13.375
 P2= 1.875
 P3= 2
 I4= .2
 V4=-5
 OA=-1.83546179E-03
 IC= 5.23718431E-03
 A= 1.5

V4=-14
 P1= 13.375
 P2= 1.875
 P3= 2
 I4=-.0714286
 V4=-14
 OA=-1.83546179E-03
 IC= 1.8704E-03
 A= 1.5

V4=-9
 O1=-.0305485195
 P1= 13.375
 P2= 1.875
 P3= 2
 I4= .111111111
 V4=-9
 OA=-1.83546179E-03
 IC= 2.90954684E-03
 A= 1.5

V4=-4
 O1=-.0551139837
 P1= 13.375
 P2= 1.875
 P3= 2
 I4= .25
 V4=-4
 OA=-1.83546179E-03
 IC= 6.54648039E-03
 A= 1.5

V4=-13
 O1=-.0108961481
 P1= 13.375
 P2= 1.875
 P3= 2
 I4= .0769230769
 V4=-13
 OA=-1.83546179E-03
 IC= 2.01430166E-03
 A= 1.5

V4=-8
 O1=-.0354616123
 P1= 13.375
 P2= 1.875
 P3= 2
 I4= .125
 V4=-8
 OA=-1.83546179E-03
 IC= 3.2732402E-03
 A= 1.5

V4=-3
 O1=-.0600270766
 P1= 13.375
 P2= 1.875
 P3= 2
 I4= .333333333
 V4=-3
 OA=-1.83546179E-03
 IC= 8.72864053E-03
 A= 1.5

V4=-12
 O1=-.0158092409
 P1= 13.375
 P2= 1.875
 P3= 2
 I4= .0833333333
 V4=-12
 OA=-1.83546179E-03
 IC= 2.18216013E-03
 A= 1.5

V4=-7
 O1=-.0403747052
 P1= 13.375
 P2= 1.875
 P3= 2
 I4= .142857143
 V4=-7
 OA=-1.83546179E-03
 IC= 3.74084594E-03
 A= 1.5

V4=-2
 O1=-.0649401
 P1= 13.375
 P2= 1.875
 P3= 2
 I4=-.50
 V4=-2
 OA=-1.83547179E-03
 IC= 1.309296E-02
 A= 1.5

V4=-11
 O1=-.0207223338
 P1= 13.375
 P2= 1.875
 P3= 2
 I4= .0909090909
 V4=-11
 OA=-1.83546179E-03
 IC= 2.38053832E-03
 A= 1.5

V4=-6
 O1=-.045287798
 P1= 13.375
 P2= 1.875
 P3= 2
 I4= .166666667
 V4=-6
 OA=-1.83546179E-03
 IC= 4.36432026E-03
 A= 1.5

V4=-1
 O1=-.0698532623
 P1= 13.375
 P2= 1.875
 P3= 2
 I4= 1
 V4=-1
 OA=-1.83546179E-03
 IC= .0261859216
 A= 1.5

V4= 15
D1=-.148462748
P1= 13.375
P2= 1.875
P3= 2
I4=-.0666666667
V4= 15
DA=-1.83546179E-03
IC=-1.7457281E-03
A= 1.5

V4= 10
D1=-.123897284
P1= 13.375
P2= 1.875
P3= 2
I4=-.1
V4= 10
DA=-1.83546179E-03
IC=-2.61859216E-03
A= 1.5

V4= 5
D1=-.0993318193
P1= 13.375
P2= 1.875
P3= 2
I4=-.2
V4= 5
DA=-1.83546179E-03
IC=-5.23718431E-03
A= 1.5

V4= 14
D1=-.143549655
P1= 13.375
P2= 1.875
P3= 2
I4=-.0714285714
V4= 14
DA=-1.83546179E-03
IC=-1.87042297E-03
A= 1.5

V4= 9
D1=-.118984191
P1= 13.375
P2= 1.875
P3= 2
I4=-.111111111
V4= 9
DA=-1.83546179E-03
IC=-2.90954684E-03
A= 1.5

V4= 4
D1=-.0944187265
P1= 13.375
P2= 1.875
P3= 2
I4=-.25
V4= 4
DA=-1.83546179E-03
IC=-6.54648039E-03
A= 1.5

V4= 13
D1=-.138636562
P1= 13.375
P2= 1.875
P3= 2
I4=-.0769230769
V4= 13
DA=-1.83546179E-03
IC=-2.01430166E-03
A= 1.5

V4= 8
D1=-.114071098
P1= 13.375
P2= 1.875
P3= 2
I4=-.125
V4= 8
DA=-1.83546179E-03
IC=-3.2732402E-03
A= 1.5

V4= 3
D1=-.0895056336
P1= 13.375
P2= 1.875
P3= 2
I4=-.333333333
V4= 3
DA=-1.83546179E-03
IC=-8.72864053E-03
A= 1.5

V4= 12
D1=-.133723469
P1= 13.375
P2= 1.875
P3= 2
I4=-.0833333333
V4= 12
DA=-1.83546179E-03
IC=-2.18216013E-03
A= 1.5

V4= 7
D1=-.109158005
P1= 13.375
P2= 1.875
P3= 2
I4=-.142857143
V4= 7
DA=-1.83546179E-03
IC=-3.74084594E-03
A= 1.5

V4= 2
D1=-.0845925408
P1= 13.375
P2= 1.875
P3= 2
I4=-.5
V4= 2
DA=-1.83546179E-03
IC=-.0130929608
A= 1.5

V4= 11
D1=-.128810376
P1= 13.375
P2= 1.875
P3= 2
I4=-.0909090909
V4= 11
DA=-1.83546179E-03
IC=-2.38053832E-03
A= 1.5

V4= 6
D1=-.104244912
P1= 13.375
P2= 1.875
P3= 2
I4=-.1666666667
V4= 6
DA=-1.83546179E-03
IC=-4.36432026E-03
A= 1.5

V4= 1
D1=-.079679448
P1= 13.375
P2= 1.875
P3= 2
I4=-1
V4= 1
DA=-1.83546179E-03
IC=-.0261859216
A= 1.5

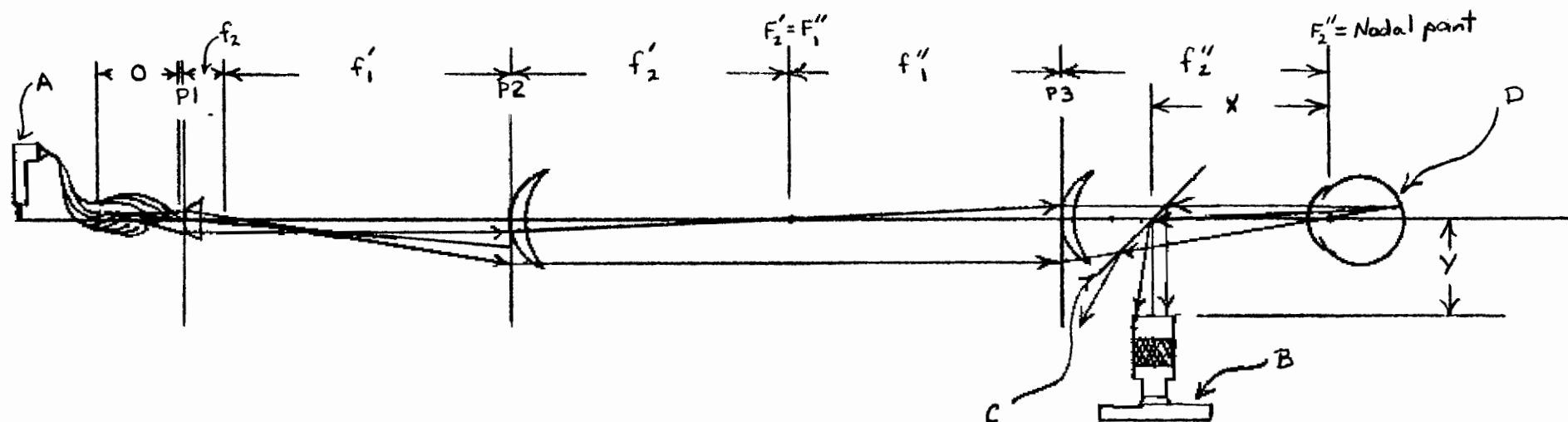


Fig. 1. Laboratory Prototype Design of the TECBP System

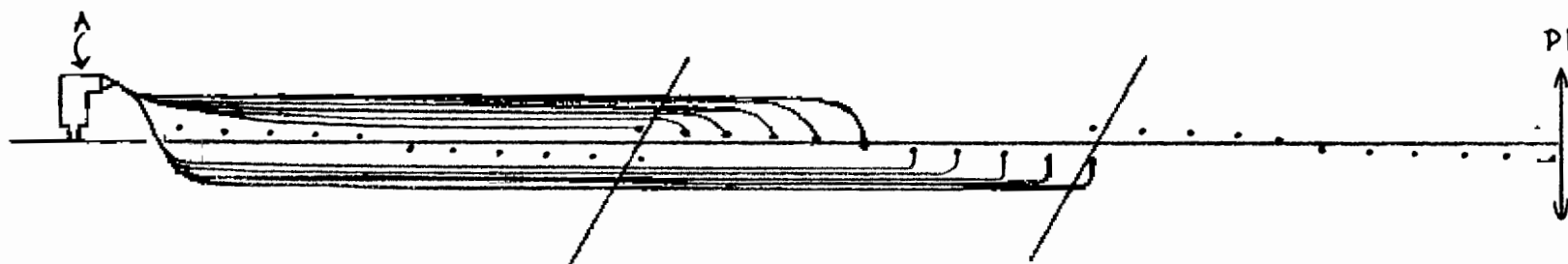


Fig. 2. Expand Diagram of Object Array

Legend

- A. Electronic Flash
- B. Photographic recording SLR camera
- C. Half silvered mirror
- D. Globe 2x scale
- E. $X + Y = f_2''$
- F. Upper scale 1mm = 1.14cm Lower scale 1cm = 1.13cm